Exercise 1.90

Gold is alloyed (mixed) with other metals to increase its hardness in making jewelry. (**a**) Consider a piece of gold jewelry that weighs 9.85 g and has a volume of 0.675 cm^3. The jewelry contains only gold and silver, which have densities of 19.3 and 10.5 g/cm^3, respectively. If the total volume of the jewelry is the sum of the volumes of the gold and silver that it contains, calculate the percentage of gold (by mass) in the jewelry. (**b**) The relative amount of gold in an alloy is commonly expressed in units of carats. Pure gold is 24 carat, and the percentage of gold in an alloy is given as a percentage of this value. For example, an alloy that is 50% gold is 12 carat. State the purity of the gold jewelry in carats.

Solution

Part (a)

The mass of gold and silver must add up to the total, which is 9.85 g. Similarly, the volume of gold and silver must add up to the total, which is 0.675 cm^3.

$$\left.\begin{aligned} m_{\text{Gold}} + m_{\text{Silver}} &= 9.85 \\ V_{\text{Gold}} + V_{\text{Silver}} &= 0.675 \end{aligned}\right\} \tag{1}$$

The density ρ is mass m divided by volume V.

$$\rho = \frac{m}{V}$$

Solve for V.

$$V = \frac{m}{\rho}$$

Substitute this result into equation (1).

$$\left.\begin{aligned} m_{\text{Gold}} + m_{\text{Silver}} &= 9.85 \\ \frac{m_{\text{Gold}}}{\rho_{\text{Gold}}} + \frac{m_{\text{Silver}}}{\rho_{\text{Silver}}} &= 0.675 \end{aligned}\right\}$$

Substitute the densities of gold (19.3 g/cm^3) and silver (10.5 g/cm^3).

$$\left.\begin{aligned} m_{\text{Gold}} + m_{\text{Silver}} &= 9.85 \\ \frac{m_{\text{Gold}}}{19.3} + \frac{m_{\text{Silver}}}{10.5} &= 0.675 \end{aligned}\right\}$$

Solve this first equation for m_{Gold}

$$m_{\text{Gold}} = 9.85 - m_{\text{Silver}} \tag{2}$$

and plug it into the second one.

$$\frac{9.85 - m_{\text{Silver}}}{19.3} + \frac{m_{\text{Silver}}}{10.5} = 0.675$$

Solve for the mass of silver.

$$\frac{9.85 - m_{\text{Silver}}}{19.3} + \frac{m_{\text{Silver}}}{10.5} = 0.675$$

$$\frac{9.85}{19.3} - \frac{m_{\text{Silver}}}{19.3} + \frac{m_{\text{Silver}}}{10.5} = 0.675$$

$$\frac{9.85}{19.3} + m_{\text{Silver}}\left(-\frac{1}{19.3} + \frac{1}{10.5}\right) = 0.675$$

$$m_{\text{Silver}}\left(-\frac{1}{19.3} + \frac{1}{10.5}\right) = 0.675 - \frac{9.85}{19.3}$$

$$m_{\text{Silver}} = \frac{0.675 - \frac{9.85}{19.3}}{\left(-\frac{1}{19.3} + \frac{1}{10.5}\right)} \approx 3.79 \text{ g}$$

From equation (2), then,

$$\begin{aligned} m_{\text{Gold}} &= 9.85 - m_{\text{Silver}} \\ &\approx 6.06 \text{ g}. \end{aligned}$$

Therefore, the percentage of gold in the jewelry is

$$\begin{aligned} \text{Percentage of Gold} &= \frac{\text{Mass of Gold}}{\text{Total Mass}} \times 100\% \\ &= \frac{m_{\text{Gold}}}{m_{\text{Gold}} + m_{\text{Silver}}} \times 100\% \\ &\approx \frac{6.06}{9.85} \times 100\% \\ &\approx 61.5\%. \end{aligned}$$

Part (b)

The purity of gold is 61.5% of 24 carat.

$$(0.615)(24 \text{ carat}) \approx 15 \text{ carat}$$